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Conservation of the nonlinear curvature perturbation in generic single-field inflation

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It is known that the curvature perturbation on uniform energy density (or comoving or uniform Hubble) slices on superhorizon scales is conserved to full nonlinear order if the pressure is only a function of the energy density (ie, if the perturbation is purely adiabatic), independent of the gravitational theory. Here we explicitly show that the same conservation holds for a universe dominated by a single scalar field provided that the field is in an attractor regime, for a very general class of scalar field theories. However, we also show that if the scalar field equation contains a second time derivative of the metric, as in the case of the Galileon (or kinetic braiding) theory, one has to invoke the gravitational field equations to show the conservation.

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I. INTRODUCTION

The CMB anisotropy observation by WMAP clearly showed us that the primordial curvature perturbations are nearly scale invariant and their statistics is Gaussian to very high accuracy [1]. This is perfectly in agreement with the predictions of the standard, canonical single-field, slow-roll inflation. Nevertheless, an indication was found in the CMB data as well as in the large scale structure data that there may be a detectable level of non-Gaussianity in the curvature perturbation. Consequently a lot of attention has been paid in recent years to possible non-Gaussian features in the primordial curvature perturbation from inflation [2]. (See articles in a focus section in CQG [3] and references therein for recent developments.) Apparently this direction of research involves nonlinear cosmological perturbations.

There are mainly two approaches to nonlinear cosmological perturbations. One is the standard perturbative approach [4–7]. This is basically straightforward and can in principle deal with most general situations as long as the perturbation expansion is applicable. But the equations can become very much involved and quite often the physical transparency may be lost.

The other is the gradient expansion approach [8–28]. In this approach, the field equations are expanded in powers of spatial gradients. Hence this is applicable only to perturbations on superhorizon scales. Nevertheless, it has a big advantage that the full nonlinear effects are taken into account at each order of the gradient expansion. At leading order in gradient expansion, it corresponds to the separate universe approach [29]. Namely, the field equations become ordinary differential equations with respect to time, hence the physical quantities at each spatial point (where 'each point' corresponds to a Hubble horizon size region) evolve in time independently from those at the rest of the space.

One of the most important results obtained in the gradient expansion approach is that the full nonlinear curvature perturbation on unifrom energy density (or comoving or uniform Hubble) slices is conserved at leading order in gradient expansion if the pressure is only a function of the energy density [27], or the perturbation is purely adiabatic. This is shown using the energy conservation law, without using the Einstein equations. Thus, without solving the field equations, one can predict the spectrum and the statistics of the curvature perturbation at horizon re-entry during the late radiation or matter-dominated era once one knows these properties of the curvature perturbaiton at horizon exit during inflation.

However, the assumption that the pressure is only a function of the energy density is not rigorously true in the case of a scalar field. It is only approximately true in the limit of the slow-roll inflation. In other words, the pressure and energy density perturbations are not 'adiabatic' in the sense of the stardard fluid dynamics. Therefore it is not completely clear exactly under which condition the nonlinear curvature perturbation is conserved in the case of a scalar field.

In this short note, we focus on a universe dominated by a single scalar field ϕ and explicitly show the conservation of the nonlinear curvature perturbation on comoving slices ($\phi = \phi(t)$) at leading order in gradient expansion. We consider a very general theory of a scalar field [30–33], including a Galileon (or kinetic braiding) field which has been attracting attention recently [34–38]. For the gravitational part we assume Einstein gravity for definiteness, but our discussion is applicable to any metric theory of gravity.

Assuming that the scalar field dynamics is in an attractor regime so that the value of the scalar field determines the dynamics completely, we find that the conservation of the nonlinear curvature perturbation holds without using the gravitational field equations just as the same as the fluid case, provided that the scalar field equation contains only first time derivatives of the metric. This condition is satisfied for a generic K-essence type scalar field, including

the case of a canonical scalar, but not for a Galileon scalar because the Galileon field equation contains second time derivatives of the metric. In the latter case, one has to invoke the gravitational field equations to see if the conservation still holds or not. In the case of Einstein gravity, the conservation is shown to hold even for a Galileon scalar field.

II. BASIC SETUP

We focus on the dynamics on superhorizon scales. We associate ϵ to each spatial derivative. So a quantity with n-th spatial derivatives will be of $O(\epsilon^n)$.

We express the metric in the $(3+1)$ form,

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = -\alpha^2 dt^2 + \gamma_{ij}(dx^i + \beta^i dt)(dx^j + \beta^j dt)\,, \tag{2.1}$$

where α, β^i and γ_{ij} are the lapse function, the shift vector, and the spatial metric, respectively. We choose the spatial coordinates such that $\beta^i = O(\epsilon^3)$. We further decompose the spatial metric as

$$\gamma_{ij} = a^2(t)e^{2\psi(t,x^k)}\tilde{\gamma}_{ij}(t,x^k)\,, \quad \det\tilde{\gamma}_{ij} = 1\,, \tag{2.2}$$

where $a(t)e^{\psi(t,x^k)}$ is the scale factor at each local point while $a(t)$ is the scale factor of a fiducial homogeneous universe. When the gradient expansion is applied to an inflationary stage of the universe, it is known that we have $\partial_t\tilde{\gamma}_{ij} = O(\epsilon^2)$ [23, 24, 27]. Then at leading order in gradient expansion, we identify ψ as the nonlinear curvature perturbation [27].

We consider a theory with the action,

$$\begin{aligned} S &= S_g + S_\phi\,; \\ &S_g = \int d^4x\sqrt{-g}\frac{1}{2\kappa^2}R\,, \\ &S_\phi = \int d^4x\sqrt{-g}\left[W(X,\phi) - G(X,\phi)\Box\phi\right]\,; \quad X \equiv -\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi\,, \end{aligned} \tag{2.3}$$

where W and G are arbitrary functions of X and ϕ. For simplicity and definiteness, we consider Einstein gravity, with $\kappa^2 = 8\pi G$, but our discussion below can be easily extended to any metric theory of gravity. The energy momentum tensor of scalar field is given by

$$T_{\mu\nu} = -2\frac{\delta S_\phi}{\delta g^{\mu\nu}} = W_{,X}\partial_\mu\phi\partial_\nu\phi + Wg_{\mu\nu} - G_{,X}\Box\phi\partial_\mu\phi\partial_\nu\phi + g_{\mu\nu}G_{,\rho}g^{\rho\sigma}\partial_\sigma\phi - (G_{,\mu}\partial_\nu\phi + G_{,\nu}\partial_\mu\phi)\,. \tag{2.4}$$

Our theory includes a Galileon scalar if $G \neq 0$ [34, 35]. If we consider the case,

$$G(X,\phi) = 0\,, \tag{2.5}$$

but retain a generic W, it corresponds to a K-essential scalar [30, 31], which includes a DBI model [32, 33] as a special case. If we consider the case,

$$G(X,\phi) = 0\,, \quad W(X,\phi) = X - V(\phi)\,, \tag{2.6}$$

we recover the conventional canonical scalar theory.

III. SCALAR FIELD EQUATION

To derive the scalar field equation, we take the variation of S_ϕ with respect to ϕ,

$$\begin{aligned} \delta S &= \int d^4x\sqrt{-g}\left[W_{,X}\delta X + W_{,\phi}\delta\phi - (G_{,X}\delta X + G_{,\phi}\delta\phi)\Box\phi - G\Box\delta\phi\right] \\ &= \int d^4x\sqrt{-g}\left[(W_{,\phi} - G_{,\phi}\Box\phi)\delta\phi - (W_{,X} - G_{,X}\Box\phi)\nabla_\mu\delta\phi\nabla^\mu\phi + (G_{,X}\nabla^\mu X + G_{,\phi}\nabla^\mu\phi)\nabla_\mu\delta\phi\right] \\ &= \int d^4x\sqrt{-g}\left[(W_{,\phi} - G_{,\phi}\Box\phi) + \nabla_\mu\left[(W_{,X} - G_{,X}\Box\phi)\nabla^\mu\phi\right] - \nabla_\mu(G_{,X}\nabla^\mu X + G_{,\phi}\nabla^\mu\phi)\right]\delta\phi\,, \end{aligned} \tag{3.1}$$

where we have used the identity,

$$Q\,\delta X = -Q\,g^{\mu\nu}(\partial_\mu\delta\phi)\partial_\nu\phi = -\partial_\mu(Q\,g^{\mu\nu}\delta\phi\partial_\nu\phi) + \partial_\mu(Q\,g^{\mu\nu}\partial_\nu\phi)\delta\phi\,, \tag{3.2}$$

and dropped the surface term after integration by part. Thus the field equation is given by

$$\frac{1}{\sqrt{-g}}\partial_\mu\Big(\sqrt{-g}\Big[(W_{,X}-G_{,\phi})\nabla^\mu\phi-G_{,X}(\Box\phi\nabla^\mu\phi+\nabla^\mu X)\Big]\Big)+W_{,\phi}-G_{,\phi}\Box\phi=0\,. \tag{3.3}$$

We introduce a unit timelike vector n^μ orthonormal to the $t=$ constant hypersurfaces,

$$n^\mu=\frac{1}{\alpha}(1,0,0,0)\,. \tag{3.4}$$

For this vector, the expansion K is given by[1]

$$K\equiv n^\mu{}_{;\mu}=\frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}n^\mu)=\frac{1}{\sqrt{\gamma}}\partial_\tau\sqrt{\gamma}\,. \tag{3.5}$$

where τ is the proper time of an observer with the 4-velocity n^μ,

$$d\tau=\alpha(t,x^k)\,dt\,. \tag{3.6}$$

Since we concentrate on the superhorizon behaviour, we neglect the spatial derivatives in (3.3) to obtain

$$-\frac{1}{\sqrt{\gamma}}\partial_\tau\Big(\sqrt{\gamma}\Big[(W_{,X}-G_{,\phi})\partial_\tau\phi-G_{,X}(\Box\phi\partial_\tau\phi+\partial_\tau X)\Big]\Big)+W_{,\phi}-G_{,\phi}\Box\phi=0\,, \tag{3.7}$$

where

$$\Box\phi=-\frac{1}{\sqrt{\gamma}}\partial_\tau(\sqrt{\gamma}\partial_\tau\phi)=-\partial_\tau^2\phi-K\partial_\tau\phi\,. \tag{3.8}$$

Then the above can be rewritten as

$$\begin{aligned}&\Big(W_{,X}-2G_{,\phi}+2G_{,X}K\partial_\tau\phi\Big)\partial_\tau^2\phi+(\partial_\tau G_{,X})K(\partial_\tau\phi)^2+G_{,X}(\partial_\tau K+K^2)(\partial_\tau\phi)^2\\&+\Big[\partial_\tau(W_{,X}-G_{,\phi})+K(W_{,X}-2G_{,\phi})\Big]\partial_\tau\phi-W_{,\phi}=0\,.\end{aligned} \tag{3.9}$$

Here an important comment is in order. In the Galileon case where $G_{,X}\neq 0$, the scalar field equation contains the second time derivative of the metric, $\partial_\tau K+K^2=\partial_\tau^2\sqrt{\gamma}/\sqrt{\gamma}$. As we shall see below, the conservation of the nonlinear curvature perturbation can be proved only if the field equation contains only first time derivatives of the metric. Thus one has to use the gravitational field equations (the Einstein equations in the present case) to eliminate the second or higher derivative terms.

The relevant components of the Einstein equations are

$$K^2=3\kappa^2E\,,\quad \partial_\tau K=-\frac{3}{2}\kappa^2(E+W+g^{\mu\nu}G_{,\mu}\phi_{,\nu})=-\frac{3}{2}\kappa^2(E+W-\partial_\tau G\partial_\tau\phi)\,, \tag{3.10}$$

or

$$\partial_\tau K+K^2=\frac{3}{2}\kappa^2\left(E-W+\partial_\tau G\partial_\tau\phi\right)\,, \tag{3.11}$$

where $E=T_{\mu\nu}n^\mu n^\nu$. Keeping only the time derivatives, E is given by

$$E=\frac{1}{\alpha^2}T_{00}=\Big[W_{,X}-G_{,X}(\Box\phi)\Big](\partial_\tau\phi)^2-\partial_\tau G\partial_\tau\phi-W\,. \tag{3.12}$$

This gives

$$\partial_\tau K+K^2\;=\;\frac{3}{2}\kappa^2\left(\Big[W_{,X}-G_{,X}(\Box\phi)\Big](\partial_\tau\phi)^2-2W\right)$$

[1] Note the change of notation from [17], in which K is defined by the 'minus' of the expansion; $K=-n^\mu{}_{;\mu}$.

$$= \frac{3}{2}\kappa^2 \left[\Big(W_{,X} + G_{,X} K \partial_\tau \phi + G_{,X} \partial_\tau^2 \phi \Big) (\partial_\tau \phi)^2 - 2W \right] . \tag{3.13}$$

Eliminating $\partial_\tau K + K^2$ in (3.9) by using Eq. (3.11), we obtain the scalar field equation involving only first time derivatives of the metric,

$$\begin{aligned}
&(W_{,X} - 2G_{,\phi} + 2G_{,X} K \partial_\tau \phi)\partial_\tau^2 \phi + (G_{,XX}\partial_\tau^2 \phi + G_{,X\phi})K(\partial_\tau \phi)^3 \\
&+ \Big[(W_{,XX} - G_{,\phi X})\partial_\tau^2 \phi + W_{,X\phi} - G_{,\phi\phi} \Big] (\partial_\tau \phi)^2 + K(W_{,X} - 2G_{,\phi})\partial_\tau \phi - W_{,\phi} \\
&+ \frac{3}{2}\kappa^2 G_{,X} (\partial_\tau \phi)^2 \Big[(W_{,X} + G_{,X} K \partial_\tau \phi + G_{,X} \partial_\tau^2 \phi)(\partial_\tau \phi)^2 - 2W \Big] = 0 .
\end{aligned} \tag{3.14}$$

IV. CONSERVATION LAW

Quite generally a conservation law corresponds to an integral of motion. This implies that it will be necessary for the scalar field equation to be effectively first order in time derivatives in order to derive a conservation law. In the present case, to show the conservation of the nonlinear curvature perturbation, we assume that the system has evolved into an attractor stage so that the time derivative of the scalar field has become a function of ϕ,

$$\partial_\tau \phi = f(\phi) . \tag{4.1}$$

Note that this may be regarded as a generalization of the slow-roll case. In this regime, the functions G and W become functions of ϕ only:

$$G = G(X, \phi) = G\left(\frac{1}{2} f^2, \phi \right) , \quad W = W(X, \phi) = W\left(\frac{1}{2} f^2, \phi \right) . \tag{4.2}$$

We also assume $f \neq 0$. This implies that ϕ can be used to determine the time slicing if desired.

We can rewrite Eq. (3.14) as

$$\begin{aligned}
&(W_{,X} - 2G_{,\phi} + 2G_{,X} K f) f_{,\phi} f + (G_{,XX} f_{,\phi} f + G_{,X\phi}) K f^3 \\
&+ \Big[(W_{,XX} - G_{,\phi X}) f_{,\phi} f + W_{,X\phi} - G_{,\phi\phi} \Big] f^2 + K(W_{,X} - 2G_{,\phi}) f - W_{,\phi} \\
&+ \frac{3}{2}\kappa^2 G_{,X} f^2 \Big[(W_{,X} + G_{,X} K f + G_{,X} f_{,\phi} f) f^2 - 2W \Big] = 0 .
\end{aligned} \tag{4.3}$$

This equation can be arranged in the form, $A(\phi) + KB(\phi) = 0$, or

$$-K = \frac{A(\phi)}{B(\phi)} , \tag{4.4}$$

where A and B are given by

$$\begin{aligned}
A(\phi) &= (W_{,X} - 2G_{,\phi}) f_{,\phi} f + \Big[(W_{,XX} - G_{,\phi X}) f_{,\phi} f + (W_{,X\phi} - G_{,\phi\phi}) \Big] f^2 - W_{,\phi} \\
&\quad + \frac{3}{2}\kappa^2 G_{,X} f^2 \Big[(W_{,X} + G_{,X} f_{,\phi} f) f^2 - 2W \Big] , \qquad (4.5) \\
B(\phi) &= 2G_{,X} f^2 f_{,\phi} + (G_{,XX} f_{,\phi} f + G_{,X\phi}) f^3 + (W_{,X} - 2G_{,\phi}) f + \frac{3}{2}\kappa^2 (G_{,X})^2 f^5 . \qquad (4.6)
\end{aligned}$$

Now we recall that K is expressed in terms of the metric components as

$$K = \frac{\partial_\tau \sqrt{\gamma}}{\sqrt{\gamma}} = \frac{3}{\alpha} \left(H + \dot{\psi} \right) , \tag{4.7}$$

where $H \equiv \dot{a}/a$ and $\dot{} = \partial/\partial t$. Integrating K along the integral curve of n^μ, that is, along $x^k =$ constant, from $t_{\rm i}$ to t, we obtain

$$\int_{t_{\rm i}}^{t} dt' \alpha K = 3 \left[\ln \left(\frac{a(t)}{a(t_{\rm i})} \right) + \psi(t, x^k) - \psi(t_{\rm i}, x^k) \right] . \tag{4.8}$$

On the other hand, the integral of the right-hand side of Eq. (4.4) gives

$$\int_{t_{\rm i}}^{t} dt' \alpha \frac{A(\phi)}{B(\phi)} = \int_{\phi_{\rm i}}^{\phi} d\phi' \frac{A(\phi')}{f(\phi')B(\phi')} = F(\phi) - F(\phi_{\rm i})\,. \tag{4.9}$$

Therefore, combining Eqs. (4.8) and (4.9), we find

$$-3\left[\ln\left(\frac{a(t)}{a(t_{\rm i})}\right) + \psi(t,x^k) - \psi(t_{\rm i},x^k)\right] = F(\phi) - F(\phi_{\rm i})\,, \tag{4.10}$$

for any t.

So far, we have not specified the time slicing. Now let us choose the uniform ϕ slicing, $\phi\big(\tau(t,x^k),x^k\big) = \phi(t)$, or regard ϕ as a time coordinate. That is, we choose the comoving slicing where $n_\mu T^\mu{}_i = 0$. In this case, the equation $\phi(t)$ satisfies becomes identical to the one for the homogeneous and isotropic universe, and the fiducial scale factor $a(t)$ can be chosen to be the one for this homogeneous and isotropic universe.

Here it is worth mentioning another particular nature of the Galileon field. An explicit expression for $n_\mu T^\mu{}_i$ at lowest order in the gradient expansion is

$$n_\mu T^\mu{}_i = \partial_\tau\phi\Big[\big(W_{,X} + KG_{,X}\partial_\tau\phi - 2G_{,\phi}\big)\partial_i\phi - G_{,X}\partial_\tau\phi\,\partial_i(\partial_\tau\phi)\Big]\,. \tag{4.11}$$

Because of the presence of the term proportional to $\partial_i(\partial_\tau\phi)$, it is clear that the uniform ϕ slicing does not necessarily coincide with the comoving slicing in general. However, in the present case, we have assumed that the system is in an attractor regime where $\partial_\tau\phi$ has become a function of ϕ alone, as given by Eq. (4.1). Therefore we have $\partial_i(\partial_\tau\phi) = \partial_i f = f_{,\phi}\partial_i\phi$. That is, in the Galileon case, the uniform ϕ slicing coincides with the comoving slicing provided that the sytem is in an attractor regime.

Then Eq. (4.10) implies

$$-3\ln\left(\frac{a(t)}{a(t_{\rm i})}\right) = F(\phi) - F(\phi_{\rm i})\,, \quad \psi_c(t,x^k) = \psi_c(t_{\rm i},x^k)\,, \tag{4.12}$$

where ψ_c is ψ evaluated on comoving slices. This is a proof of the conservation of the nonlinear curvature perturbation on comoving slices. The key for the proof is the attractor behaviour of the scalar field, Eq. (4.1).

V. CONCLUSION

We have shown that the nonlinear curvature perturbation on comoving slices is conserved on superhorizon scales for a very general class of single-field inflation. It can be derived by using only the scalar field equation if it contains only first derivatives of the metric, while the gravitational equations are necessary if it contains second or higher derivatives of the metric. The key, necessary condition is that the scalar field is in an attractor regime so that ϕ can be taken as a time coordinate.

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